Inferring the radial parameter in rotational acceleration data

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# Formulation

Consider a rigid body undergoing circular motion in the horizontal place. An accelerometer is attached to the body at a radial distance *r*  from the axis of rotation. The accelerometer will provide a time-series output

at regular time intervals . Our initial problem is to infer the value of *r* from this data. given only the accelerometer output.

# Expression involving parameter r

Consider the systemssume angular velocity at time ti, and uniform angular acceleration α during the time interval ti to ti+1.

The new angular velocity will be

and the radial accelerations are therefore

giving

where we used

(1)

# Solution via optimization methods

From the full expression, we can use minimization methods to find the optimal value for *r*, i.e. we can form a cost function

and minimize against r. We actually want the value of r that brings c closest to zero, so we will have to rectify it by an additional operation such as squaring it, eg we will minimize

( Ferenc: I pursue this line of thought in the next document; the rest can be skipped )

# Student exercises

## Direct algebraic solution

We can solve the above equation and obtain a closed-form solution for r.

r = ...

Are there any simplification opportunities? Any restrictions implied from the requirement that r must be non-negative? Can it be rewritten in a nicer form?

( Ferenc: I start exploring this a bit in the accompanying doc )

## Solution under a simplification

By ignoring[[1]](#footnote-1) the term in, this simplifies to

which can easily be solved for r.

after which alpha is easily found.

## Warm-up programming exercise for implementing a minimization problem

If alpha is known, eg. we generate data using a constant-angular-acceleration scenario, then we can implement a very simple cost function:

We can minimize c by varying r, and obtain the optimal value for r. This will validate a simple minimization algorithm, into which we can inject increasingly sophisticated cost functions.

1. The assumption is that , which implies ; ;

   i.e. the change in omega is much smaller than its current value. This assumption would seem to fail near zero speed, with large accelerations. eg a “kickstart” scenario. [↑](#footnote-ref-1)